Cone-beam tomography of propagation-based imaging

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A tomography algorithm is proposed for the reconstruction of the three-dimensional (3D) distribution of the real refractive index of a pure phase sample from the projections of cone-beam propagation-based imaging (PBI). The algorithm first retrieves the integral of the real refractive index based on the formula described in this letter, and then a conventional computed tomography algorithm is performed to reconstruct the 3D distribution of the real refractive index of the sample. The computer simulations show that this method is effective for PBI tomography. © 2008 American Institute of Physics. [DOI: 10.1063/1.3043452]

Since it was introduced in the 1970s, great advances have been made in x-ray computed tomography (XCT).1,2

Today, XCT is one of the most widely used technologies in medicine, biology, and material sciences. Conventional XCT differentiates regions within the imaged sample based on their capacity for attenuating x rays. The contrast provided by this technique is highly effective for distinguishing regions that have significant differences in x-ray attenuation. Nevertheless, many regions of interest attenuate x rays very poorly and provide images of low contrast. However, it is well known that the phase of an x ray changes more rapidly than does its amplitude when passing through these samples; therefore, it is possible to use phase information to improve imaging contrast. There are four phase contrast imaging mechanisms,3–8 x-ray interferometry imaging, diffraction-enhanced imaging, propagation-based imaging (PBI), and gratings-based imaging. In this letter, only PBI will be discussed.

PBI was introduced by Wilkins et al.5 in 1996. The experimental configuration involves a source of high spatial coherence and an x-ray imaging detector. A spherical wavefront emanated from the point source becomes distorted as it passes through the object. By recording the intensity of the wavefront at a sufficient distance from the sample, intensity variations due to sharp refractive index variations in the sample may be detected that correspond to a form of differential phase contrast imaging. Much effort has been put into reconstruction of samples generated by PBI projections. For example, Bronnikov9 provided an elegant algorithm for reconstructing parallel beam projections, while Myers et al.10 derived a paraxial cone-beam computed tomography (CT) reconstruction formula. Gureyev et al.11 proposed an algorithm for reconstruction under specified conditions of the refractive index. The algorithms developed to date for phase contrast tomography can be classified into two types: direct reconstruction and “two-step” reconstruction. The former does not require an intermediate step of phase retrieval and reconstructs the refractive index from the intensity measured in one or more planes. The two-step process first retrieves the refractive index integral and then reconstructs the refractive index by applying a conventional CT algorithm. In this letter, we provide a two-step method for reconstructing the PBI projections based on the discrete Fourier transform (DFT).12

Suppose the x-ray refractive index of a sample is \( n = 1 - \delta = i \beta \). In the geometrical optics approximation, the phase difference \( \phi \) for a ray through the sample, relative to vacuum, is given by

\[
\phi(x,y) = -k \int_l \delta(x,y,z)dz, \tag{1}
\]

where \( k = 2\pi/\lambda \) and \( l \) is the path of the ray. Then, the intensity distribution \( I \), at distance \( z \) from the sample can be derived based on the Kirchhoff formula in the Fresnel diffraction case5,8

\[
I_A(Mx,My;R,k) = \frac{I_0(x,y)}{M^2} \left[ 1 + \frac{\bar{z}}{kM} \nabla^2_{x,y} \phi(x,y) \right] \tag{2}
\]

where \( M = (R + z)/R \) is the magnification of the projection, \( R \) is the distance from the source to the sample, \( z \) is the distance between the sample and detector, and \( I_0 \) is the absorption-contrast intensity measured at \( z = 0 \). Equation (2) is valid to first order in \( (z/kM)\nabla^2_{x,y} \phi \) assumed small. In image processing, a Laplace operator is approximated by a convolution sum

\[
\nabla^2_{x,y} \phi(x,y) = [\phi(x + 1,y) - \phi(x,y)] - [\phi(x,y) - \phi(x - 1,y)] + [\phi(x,y + 1) - \phi(x,y)] - [\phi(x,y) - \phi(x,y - 1)]. \tag{3}
\]

Using the circular shifting property of the DFT, the DFT of both sides of Eq. (3) can be expressed as

\[
\text{FT}[\nabla^2_{x,y} \phi(u,v)] = \text{FT}\left[ \phi(u,v) \right] [\exp(-j2\pi u/N) + \exp(j2\pi u/N)] = \left[ \exp(j2\pi u/N) + \exp(-j2\pi u/N) \right] \left[ \exp(j2\pi v/N) - 4 \right], \tag{4}
\]

where FT denotes the Fourier transform. Simplifying Eq. (4), we obtain

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The schematic diagram of the PBI simulation. $S$ is the x-ray source. The center of the sample is placed in the origin and the plane $P_{0}$ is pressed against the sample. We simulated the intensity distribution in the plane $P_{z}$ using the intensity recorded in the plane $P_{0}$.

$$\text{FT}[\phi(u,v)] = \frac{\text{FT}[\nabla^{2}_{x,y} \phi(u,v)]}{2 \cos(2\pi u/N) + 2 \cos(2\pi v/N) - 4},$$

which in combination with Eq. (2), yields

$$\text{FT}[\phi(u,v)] = \frac{\text{FT}[(I_{0}M^{2}/I_{0}-1)kM/z]}{2 \cos(2\pi u/N) + 2 \cos(2\pi v/N) - 4}.$$  

Note that $1/[2 \cos(2\pi u/N) + 2 \cos(2\pi v/N) - 4]$ is a low-pass filter. Therefore, the high-frequency components of noisy data will be suppressed. The FT[\phi(x,y)] computed by Eq. (6) approximates the actual DFT of \phi(x,y), and the approximation in the frequency domain will influence the whole image quality. There is a singularity at point (0,0). The value at (0,0) can be computed as

$$\text{FT}[\phi(0,0)] = \sum_{(u,v) \neq (0,0)} \text{FT}[\phi(u,v)],$$

if the value of \phi(0,0) is zero. Equation (6) establishes a relationship between the DFT of \phi(x,y) and the DFT of $(I_{0}M^{2}/I_{0}-1)kM/z$. When the Fourier transform of \phi(x,y) is computed by Eq. (6), \phi(x,y) can be obtained by the inversion DFT. Integrated with a conventional cone-beam tomography algorithm, the real part of the refractive index $\delta$ can be reconstructed.

Simulated data were used to evaluate this method. To understand the simulation process, consider Fig. 1. The plane $P_{0}$ was pressed against the sample. An x ray was uniquely determined by the source and any point in plane $P_{0}$. For each pixel in plane $P_{0}$, we generated an x ray and then computed the phase and the amplitude using the ray-tracing method. According to the theory developed by Faganin et al., the intensity in the plane $P_{c}$ could be computed by using the wavefront obtained in the plane $P_{0}$. When the wavefront in the plane $P_{0}$ is obtained, the intensity in the plane $P_{z}$ is computed based on Fresnel–Kirchhoff integral.

The detector pixel size was 10 $\mu$m and the projection dimensions were $2.0 \times 2.0$ mm$^{2}$. 180 projections were generated for one projection per degree. In our experiment, the x-ray energy was 20.08 keV. The distance between source and object was 0.1 m and the image distance was 0.02 m. The phantom used for the simulation was the modified three-dimensional Shepp–Logan, which was composed of 12 ellipsoids, as illustrated in Fig. 2. Ellipsoids 1 and 4 were filled with silicon dioxide ($\delta=1.134 \times 10^{-6}$); ellipsoids 5, 8, and 10–12 were filled with coconut oil ($\delta=1.143 820 1 \times 10^{-9}$); ellipsoids 3, 6, 7, and 9 were filled with glycerin ($\delta=1.230 611 4 \times 10^{-9}$); ellipsoid 2 was filled with water ($\delta=5.720 067 4 \times 10^{-7}$).

Figure 3 describes the main idea underlying this letter. Figure 3(a) shows the original projection of the real part of the refractive index taken at 20°. Figure 3(b) is the simulated intensity distribution, and the reconstructed projection is shown in Fig. 3(c). Comparing Fig. 3(a) with Fig. 3(c), we can see that the suggested method reconstructs the projections quite well. To make a quantitative comparison, we use the profiles of the original values and the reconstructed values along the solid lines in Figs. 3(a) and 3(c). The profiles in Fig. 3(d) show that the reconstructed values are close to the original values. There are small artifacts in the reconstructed projections that are caused by the approximation in Eq. (6).

With the integral values obtained from the simulation, we reconstructed the real part of the refractive index by the well-known Feldkamp–Davis–Kress (FDK) (Ref. 15) algorithm. The overall dimensions of the reconstructed volume were $2.0 \times 2.0 \times 2.0$ mm$^{3}$.

Figure 4(a) shows the reconstructed slice at $z=0.25$ in the normalized coordinates system using the extracted $\phi(x,y)$. The origin of the normalized coordinate system is at the center of the phantom. The reconstructed slice using the original $\phi(x,y)$ is shown in Fig. 4(b). The profiles taken from the two reconstructed slices are given in Fig. 4(c). The artifacts near the edge of the objects are more serious than are the artifacts in other places because the first order in

![Image](https://via.placeholder.com/150)

**FIG. 4.** (Color online) (a) The original projection taken at 20°, (b) the simulated intensity distribution, (c) the reconstructed projection, and (d) the profiles taken from (a) (the solid line) and (c) (the dashed line).
\{ (z/kM) \nabla^2_{x,y} \phi \} \text{ neglected in Eq. (2)} \text{ is large at the edges of the objects.}

In summary, in this letter, an algorithm that reconstructs slice images of an object, based on cone-beam PBI, is proposed. The algorithm retrieves the integral of the real part of the refractive index based on the DFT, and then reconstructs that real refractive index using the FDK algorithm. Simulated projections were used to verify the validity of the algorithm. The reconstructed slices show that the algorithm is effective for cone-beam phase contrast tomography.

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